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Investigation of Fast Wave Beam-Plasma Interactions

Quarterly Report No. 5

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UNITED STATES ARMY ELECTRONICS COMMAND · FORT MONMOUTH, N.J.
CONTRACT DA 28-043 AMC-02041(E)

INSTITUTE FOR PLASMA RESEARCH
STANFORD UNIVERSITY, STANFORD, CALIFORNIA

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INVESTIGATION OF FAST WAVE BEAM/PLASMA INTERACTIONS

U. S. Army Electronics Command
Fort Monmouth, New Jersey
REPORT No. 6
CONTRACT DA-28-043-AMC 02041(E)

5TH QUARTERLY REPORT
1 March - 31 May 1967

SU-IPR Report No. 170
June 1967

Institute for Plasma Research
Stanford University
Stanford, California

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Contract DA-28-043-AMC-02041(E)

for the period

1 March - 31 May, 1967.

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ABSTRACT

This report describes a program of work on beam/plasma interaction. Both electrostatic and electromagnetic wave amplifying mechanisms are under investigation. For the former, studies in the absence of a static magnetic field are directed towards verifying the theory for the cases of finite beam/infinite plasma, and beam/surface wave amplification, when transverse modulation is applied. A dipole resonant coupling system for such interactions is under study. Two distinctly different lines are being followed for interactions in the presence of a static magnetic field: Electrostatic cyclotron harmonic wave interaction is being examined, both theoretically and experimentally, and the potentialities of electromagnetic wave growth in the "whistler" mode are being investigated.

FOREWORD

This contract represents a three-year program of research on "Fast Wave Beam/plasma Interactions" which is proceeding in the Institute for Plasma Research, Stanford University, under the direction of Prof. F. W. Crawford as Principal Investigator. The work is part of PROJECT DEFENDER and was made possible by the support of the Advanced Research Projects Agency under Order No. 695. It is conducted under the technical guidance of the U. S. Army Electronics Command. This is the fifth Quarterly Report, and covers the period 1 March to 31 May, 1967.

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I. INTRODUCTION

The wave amplification effect associated with the interaction of an electron beam and a plasma has attracted considerable attention over the last few years, particularly from microwave tube specialists to whom such interactions offer possibilities of constructing very high gain devices which should be electronically tunable over wide frequency ranges. Since the plasma plays the role of a conventional slow-wave structure, the interaction region should be free of metallic structures, a particularly significant characteristic if millimeter wave operation is envisaged.

The work being carried out under this contract is directed towards utilizing the beam/plasma amplification mechanism in microwave device applications. So far, despite the efforts of many groups, it has not been found possible to realize this potential fully. The most serious obstacles to progress are that efficient coupling of an rf signal into and out of the interaction region has been found difficult to achieve, and that the amplifiers are frequently very noisy compared with most conventional microwave tubes. The necessity of providing the means of plasma generation within the device, and the presence of a relatively high background gas pressure, add constructional problems beyond those normally encountered with vacuum tubes. Although satisfactory engineering solutions to these latter difficulties could certainly be found, the coupling and noise problems still require considerable further study to determine whether competitive devices can be developed.

Of the many widely differing aspects of beam/plasma interaction, three have been chosen for close examination under this contract. The first of these is the interaction of an electron beam with a plasma when the modulating fields, and the wave growth, are in the first azimuthally-varying mode. Since with transverse modulation several interesting interaction and coupling mechanisms become possible, it is intended that a major portion of the work under this contract should consist of a thorough investigation of such phenomena.

Most previous work has been concerned with the theoretical description and demonstration of beam/plasma interaction mechanisms that can

be derived from cold plasma theory, i.e., from theory in which it is assumed that the plasma electrons have no thermal or directed motions, and that the injected beam is monoenergetic. When a dc magnetic field is present, microscopic theory, in which single-particle behavior is followed, predicts a much wider range of amplification mechanisms. Some of these are simply modifications of those occurring in the absence of the magnetic field, while others involve interaction of beams with transverse energy with slow "cyclotron harmonic waves." This constitutes our second area of interest, i.e., that of wave growth in magnetoplasmas when the electron beam has a substantial component of transverse energy.

Our third area of interest is in electromagnetic wave amplification. Theoretical studies show that, in addition to electrostatic wave growth phenomena such as those just described, there is the possibility of obtaining appreciable growth in the "whistler" mode when an electron beam with transverse energy interacts with a magnetoplasma. This mode is a right-hand, circularly-polarized electromagnetic wave, i.e., its electric field vector rotates in the right-hand sense, which is also (conventionally) the sense of rotation of the electrons about the magnetic field lines. If a beam with transverse energy is moving along the field lines, there is consequently a possibility of energy being transferred from the electrons to the wave, and hence, for wave amplification to occur. The purpose of our work is to demonstrate this type of interaction, and to examine its potentiality for coupling to slow- and fast-wave circuits. Here "fast-wave" is interpreted to mean that the phase velocity of the wave is of the order of the velocity of light.

Previous quarterly reports (QR) have described the background for each of the topics in detail. Progress made during the reporting period will be described in the succeeding sections.

II. BEAM/PLASMA AMPLIFICATION WITH TRANSVERSE MODULATION

In previous QR, various general types of electrostatic beam/plasma interactions have been discussed in detail. The particular conditions of interest to us under this project are those of a finite beam interacting with a finite plasma surrounded by one or more dielectrics and a metallic waveguide. We may distinguish two cases: one in which the beam diameter is small compared with the plasma diameter, and one in which the beam fills the plasma. In the latter case, the interaction occurs between the beam and surface waves that may propagate along the column. Our aim is to study both types of interaction, both theoretically and experimentally, for modulation applied transverse to the beam. This mode of interaction has inherently interesting features in relation to improved coupling between the interaction region, and structures outside.

In QR 4,[1] the question of the optimum choice of plasma to waveguide radius ratio, and the maximum beam voltage for controlled mode selection ($m = 0$ or 1) were discussed, together with preliminary experimental results on input/output coupling, propagation of the amplified signal, and the appearance of ion oscillations. These studies have been pursued during the current reporting period.

(A) Theoretical Studies of Beam/plasma Interaction.

For the two beam/plasma radius ratio limits to be studied, it is expected that when the ratio is small, the spatial growth rate will be high, but that the electric field near the glass wall of the tube and in the waveguide will be weak, making strong coupling difficult to realize. The other limit, with the beam/plasma radius approaching unity, provides much stronger electric field at the dielectric interface, and therefore better coupling. Coupling Q's up to 50 have been observed in this configuration and reported in previous QR. Due to the larger interaction region, i.e. lower beam/plasma frequency for the same current, the gain is somewhat lower. Amplification in this mode may also be more strongly affected by radial plasma inhomogeneity.

In the quasistatic approximation, the relevant dispersion relation for the geometry of Fig. 1 is

$$D(\omega,\beta) = \begin{vmatrix} I_m(\beta a) & , & -I_m(\beta a) & , & -K_m(\beta a) & , & 0 & , & 0 \\ -\epsilon_{pb} I'_m(\beta a) & , & \epsilon_g I'_m(\beta a) & , & \epsilon_g K'_m(\beta a) & , & 0 & , & 0 \\ 0 & , & I_m(\beta b) & , & K_m(\beta b) & , & -I_m(\beta b) & , & -K_m(\beta b) \\ 0 & , & -\epsilon_g I'_m(\beta b) & , & -\epsilon_g K'_m(\beta b) & , & I'_m(\beta b) & , & K'_m(\beta b) \\ 0 & , & 0 & , & 0 & , & I_m(\beta c) & , & K_m(\beta c) \end{vmatrix} = 0 \tag{1}$$

where,

$$\epsilon_{pb} = 1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2}{(\omega - \beta v_b)^2} \quad , \tag{2}$$

and ϵ_g is the relative permittivity of the glass. Equation 1 can be written as,

$$1 - \frac{\omega_p^2}{\omega^2} - \frac{\omega_b^2}{(\omega - \beta v_b)^2} = F(\beta) \quad , \tag{3}$$

where,

$$F(\beta) = -\frac{\Delta_1}{\Delta_2} \quad , \tag{4}$$

and

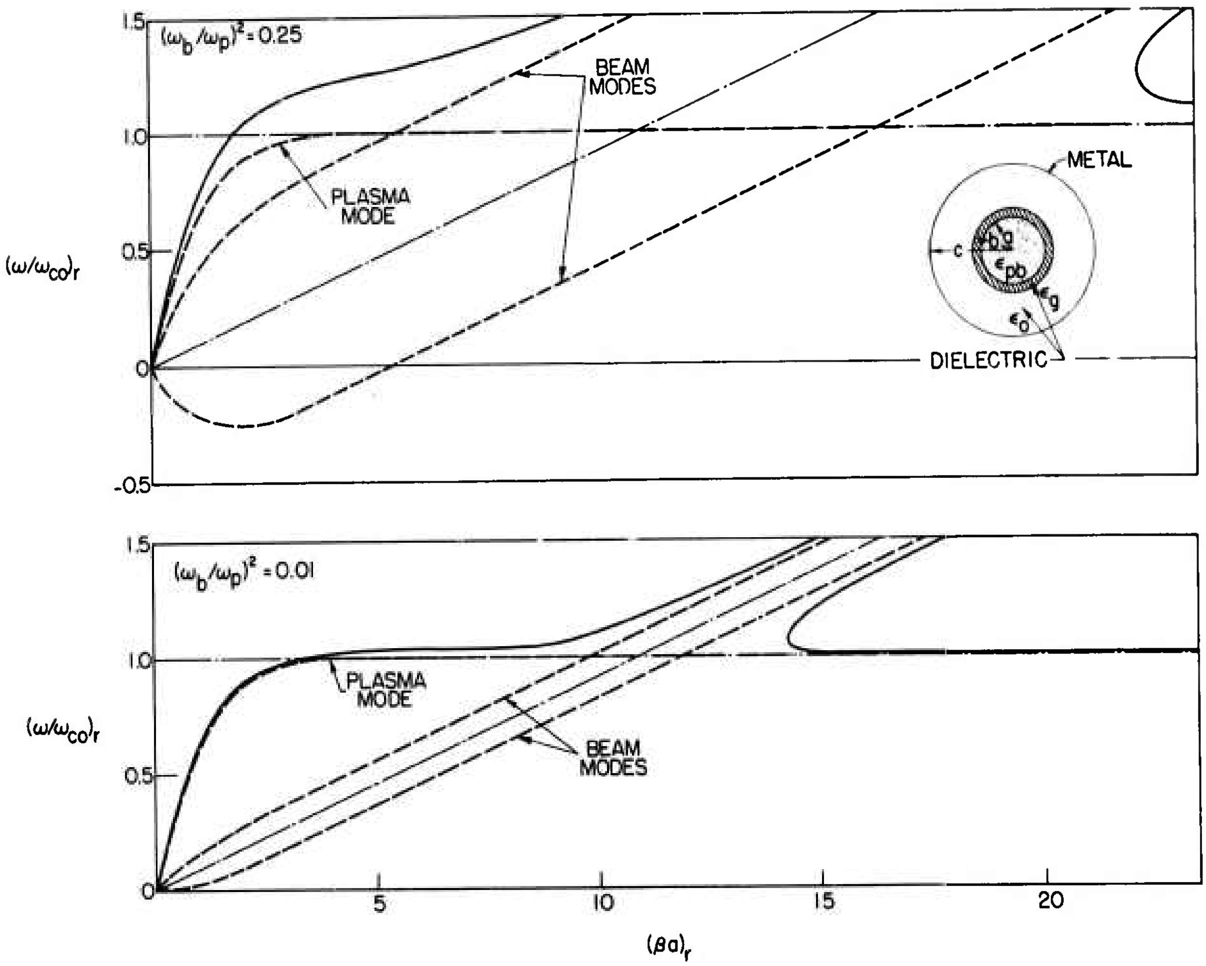

FIG. 1. Beam/plasma interaction with surface waves. Axisymmetric mode $[(b/a) = 1.18\ ,\ (c/a) = 1.36\ ,\ \epsilon_g = 4.60]$.

$$\Delta_1 = I_m(\beta a) \begin{vmatrix} \epsilon_g I_m'(\beta a) & , & \epsilon_g K_m'(\beta a) & , & 0 & , & 0 \\ I_m(\beta b) & , & K_m(\beta b) & , & -I_m(\beta b) & , & -K_m(\beta b) \\ -\epsilon_g I_m'(\beta b) & , & -\epsilon_g K_m'(\beta b) & , & I_m'(\beta b) & , & K_m'(\beta b) \\ 0 & , & 0 & , & I_m(\beta c) & , & K_m(\beta c) \end{vmatrix}$$

$$\Delta_2 = I_m'(\beta a) \begin{vmatrix} -I_m(\beta a) & , & -K_m(\beta a) & , & 0 & , & 0 \\ I_m(\beta b) & , & K_m(\beta b) & , & -I_m(\beta b) & , & -K_m(\beta b) \\ -\epsilon_g I_m'(\beta b) & , & -\epsilon_g K_m'(\beta b) & , & I_m'(\beta b) & , & K_m'(\beta b) \\ 0 & , & 0 & , & I_m(\beta c) & , & K_m(\beta c) \end{vmatrix} . \qquad (5)$$

In order to make a detailed investigation of Eq. (3), a program was first written to find the real roots of the dispersion relation. These are shown in Fig. 1. It will be seen that for the $m = 0$ mode, the fast surface wave in the beam/plasma interaction case approaches the plasma dispersion for the long wavelengths, whereas for short wavelengths it has for asymptote the fast beam surface wave. For $(\omega/\omega_{c0}) < 1$ there are two pure real roots, the second having for asymptotes the slow beam surface wave and the plasma surface wave.

Our current efforts are directed to determining the complex roots of the dispersion relation for both the $m = 0$ and $m = 1$ modes.

(B) Experimental Studies of Beam/plasma interaction.

In QR 4, computations of tube geometry such that the beam characteristic intersects the $m = 1$ plasma mode only were presented,

and it was pointed out that this could be assured in two ways: either by increasing the beam voltage, or by reducing the ratio (c/a) (see Fig. 1). During the quarter, a new beam/waveguide system has been constructed with $(c/a) = 1.56$, and designed for interaction with the $m = 1$ mode only for $V_b \geq 565$ V. This is shown in Fig. 2. Facilities have been provided for excitation of the $m = 0$ and $m = 1$ modes separately, and the input coupler can be rotated to distinguish the mode of excitation actually occurring under given conditions. One end of the tube has been made non-reflecting by provision of absorbing material and a coating of "Aquadag". The first measurements with the new system concerned the dc stability of the discharge and choice of the best operating point. It was found that even with a two-grid system, for control and acceleration of the beam, the discharge could be space-charge limited for only a short time, and that long term stability of the column could be achieved through temperature-limited operation only. The best DC operating point was found to be $V_b = 150$ V, and $(I_b + I_p) = 1$mA, for which the beam line intersects both the $m = 0$ and $m = 1$ modes, however. To ensure stability of the plasma density, a highly-stable DC heater supply is used.

Since the best operating point still allowed propagation and growth in the $m = 0$ mode, a number of measurements were made of its characteristics. It is essential to know these, and to avoid such effects as beam break-up due to noise in the $m = 0$ mode while studying growth of modulated signals in the $m = 1$ mode. Some representative data on amplitude and phase along the tube are shown in Fig. 3. It can be seen that the growth rate is quite low; less than 1 dB/cm. When the computations of the coupled roots of the dispersion relation are completed, we will be able to check the agreement between simplified theory and these experimental results.

The beam break-up phenomenon referred to manifests itself as shown in Fig. 4, by a sudden change of the amplitude of the spatially growing signal, and somewhat increased phase shift.

When the break-up is caused by the modulating signal, the break-up region moves towards the cathode as the plasma frequency is approached

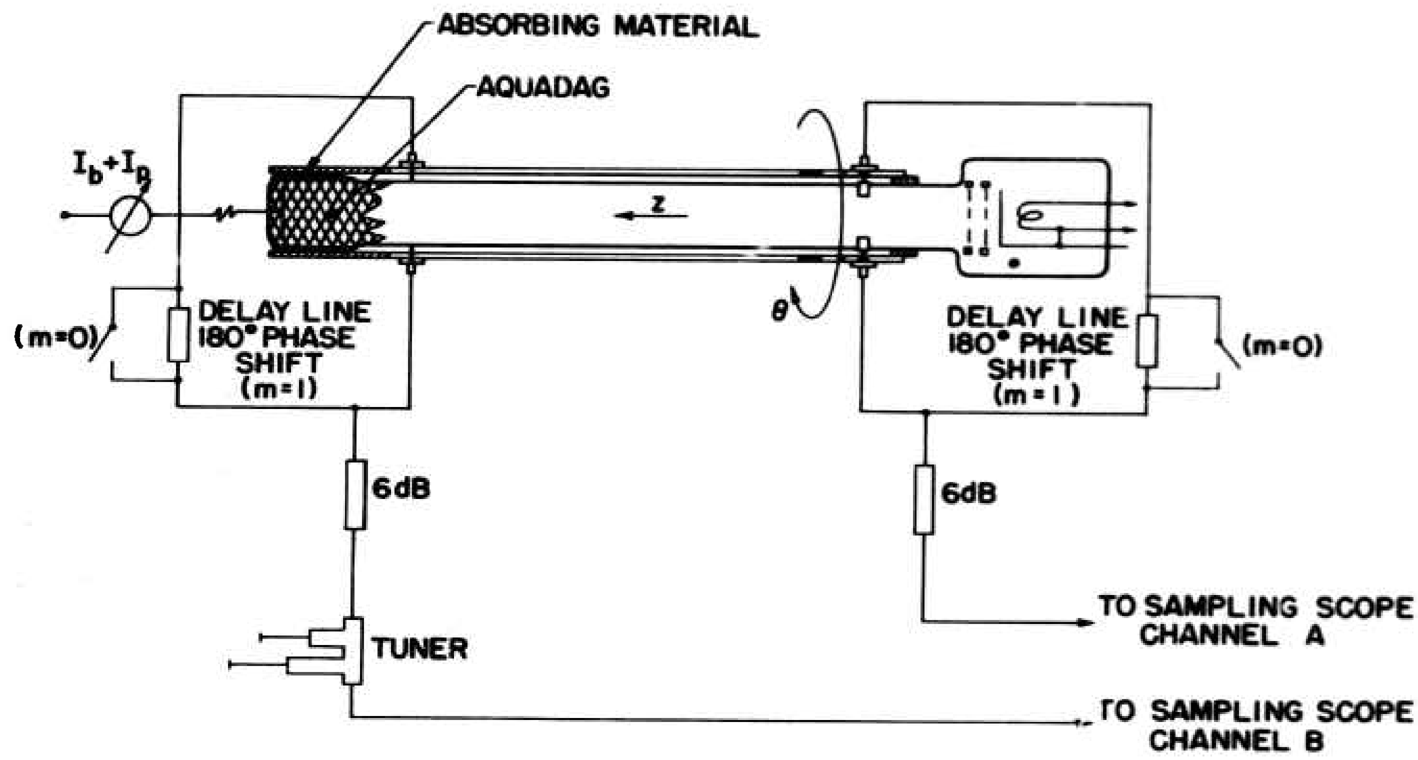

FIG. 2. **Beam/plasma interaction with surface waves. Experimental set-up.**

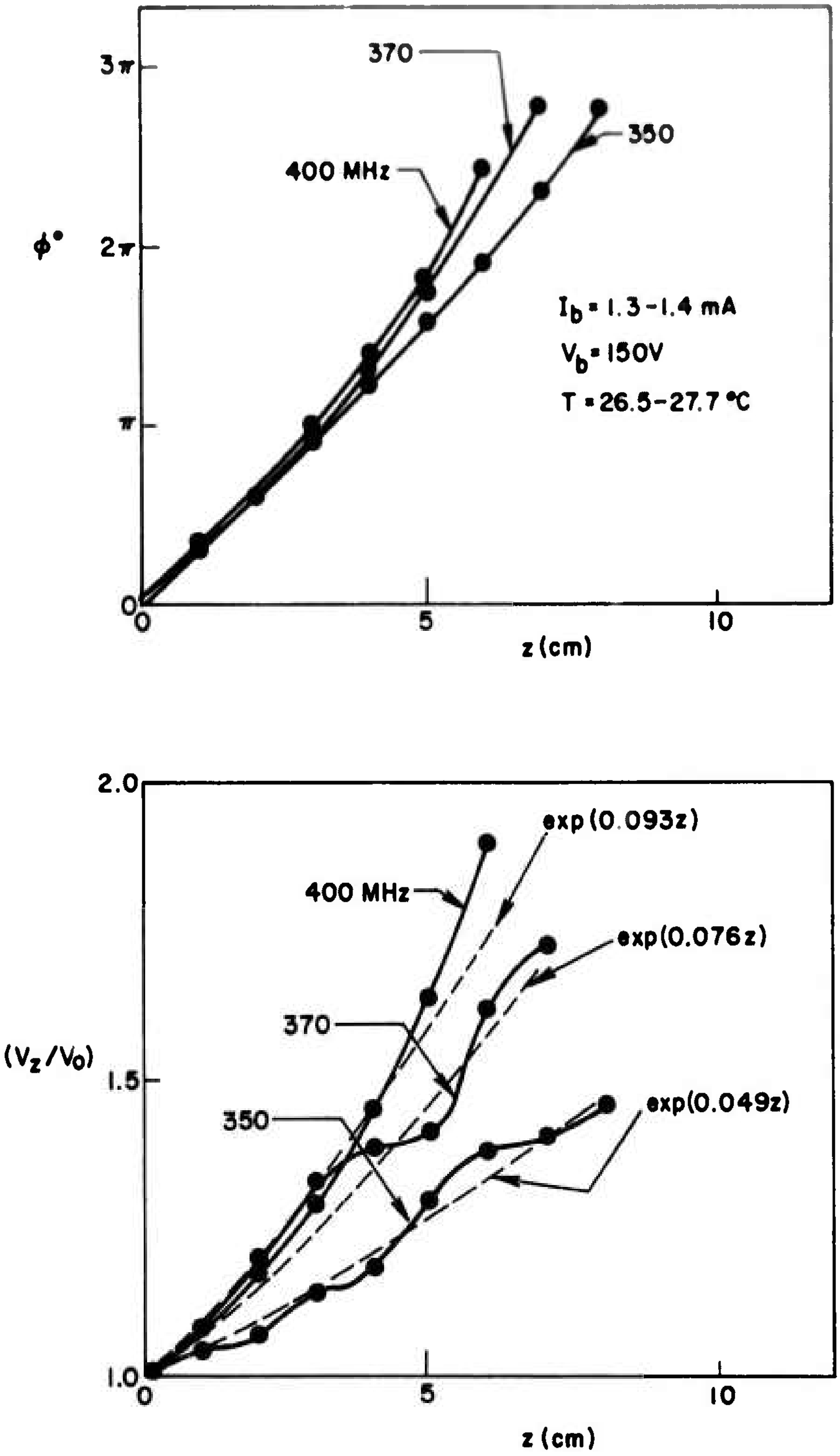

FIG. 3. Beam/plasma interaction with surface waves. Growth and phase-shift characteristics for the m = 0 mode.

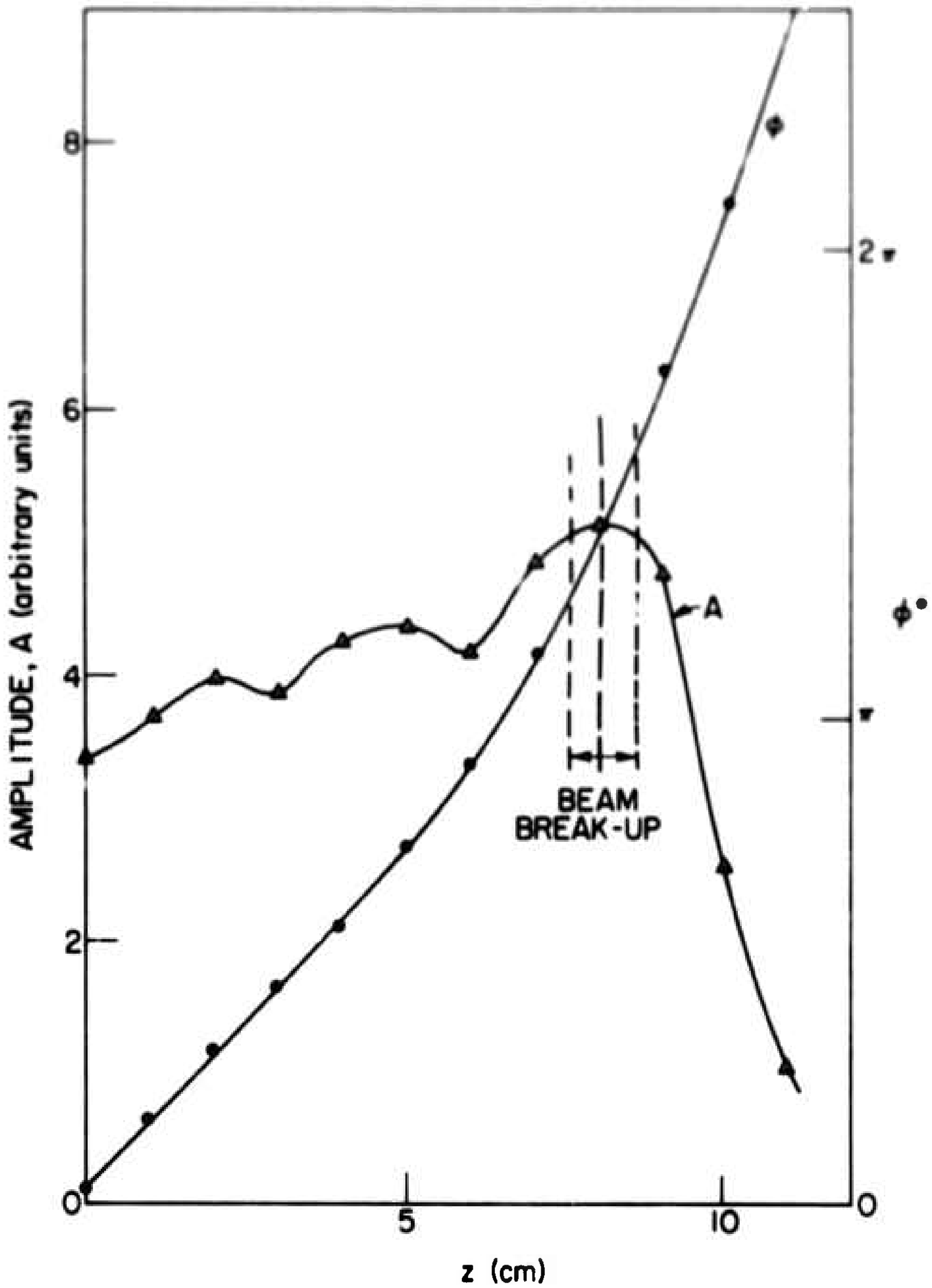

FIG. 4. Beam/plasma interaction with surface waves. Beam break-up effects.

from below. All modes are highly attenuated beyond the beam break-up region. Theoretical predictions are followed further in that increasing the beam current, or decreasing the beam voltage, moves the break-up region towards the excitation coupler.

It will be observed from Figs. 3 and 4 that the growing signals exhibit a "beating" effect which is currently under investigation. Its origin is thought to be as follows: At each frequency below the cut-off, infinitely many propagation constants could be found in principle using the dispersion relation. If we consider two of these, we have waves propagating as $A_1 \exp j(\omega t - \beta_1 z + \varphi_1)$, with complex β_1 and $A_2 \exp j(\omega t - \beta_2 z + \varphi_2)$. It is highly probable that both can be excited by our input coupler, and that what is observed experimentally is a mixed wave,

$$A_2 \exp j(\omega t - \beta_2 z + \varphi_2) + A_1 \exp j(\omega t - \beta_1 z + \varphi_1) = A \exp j\varphi \quad ,$$

which manifests itself as spatial beating. In our future work we intend to investigate this effect and its possible suppression closely.

III. ELECTROSTATIC WAVE AMPLIFICATION IN MAGNETOPLASMAS

When the beam and/or plasma have directed or thermal motions in the transverse and axial directions, it is necessary to derive the appropriate dispersion relations using a Boltzmann equation formalism. The results of doing so were discussed rather generally in QR 1 where it was pointed out that, for a high enough value of the parameter (ω_b/ω_c) , i.e., the ratio of beam plasma frequency to electron cyclotron frequency, even an ion-neutralized electron beam could be unstable, and that in the presence of a background plasma the instability threshold for the beam density could be reduced. The purpose of this project is to investigate such interactions, and to determine their potentialities for microwave applications.

Numerous theoretical predictions of the instabilities have been made at Stanford and elsewhere. Basically, the theory predicts growth in passbands centered on the electron cyclotron harmonic frequencies $(n\omega_c)$. No further computations will be carried out under this project until our experimental parameters have been measured. Those computations carried out to date are being summarized in a Ph.D. thesis being written by J. A. Tataronis.

So far, few controlled experiments have been carried out to check the theory, though observations of strong noise emissions from magnetoplasmas containing charged particles with appreciable transverse velocities provide significant support for the existence of the predicted mechanisms. The studies planned under this contract are intended to provide results under refined experimental conditions, and to put the theory on a firm quantitative basis. In particular, we wish to verify the dispersion relation for the realistic case of a delta-function beam interacting with a warm plasma.

(A) Experimental Studies

The aim of the experimental work under this project is to excite growing waves by means of an electron beam injected into the plasma, and to study the variation of the growth rate as a function of the longitudinal and transverse energies of the beam. The first, and simplest,

way of imparting transverse energy to the beam is to inject it through an increasing magnetic field into the interaction region. This does not create the delta-function transverse velocity distribution which would be most desirable for checking against theory. A more satisfactory approach is the use of a "corkscrew" injection system.[2] A third method which it was hoped to apply because of its greater flexibility is to impart transverse energy to the electrons by cyclotron heating in a small rf cavity through which the beam passes before entering the plasma region. This has been found extremely difficult to realize for our experimental conditions, however. For convenience in our initial studies, the first method has finally been adopted.

The experimental set-up is as shown in Fig. 5. Eight large coils have been spaced so as to give an axial magnetic field uniform to better than 0.5% over a region of five to six centimeters centered at the probe. To produce a rapidly varying magnetic field close to the cathode, two auxiliary coils have been provided. These reduce the field in the vicinity of the cathode, but do not disturb the uniformity of the field in the region near the probe by more than two or three percent. The fields due to the two coil systems are controlled by separate power supplies so that inhomogeneity can be varied relative to the background homogeneous field.

The theoretical predictions for this type of instability indicate that it will generally be absolute, i.e. signals will grow from noise, and external modulation will not be required to obtain an output. This has been confirmed qualitatively by the previous experimental work which it is hoped to refine quantitatively in these studies. For example, in a PIG discharge Landauer[3] observed radiation out to about the 45th harmonic, fitting the relation $(\omega/\omega_c) = n$ to better than 2%. Bekefi and Hooper[4] observed strong cyclotron harmonic radiation from a beam generated discharge in mercury-vapor. As here, they produced the necessary transverse energy by magnetic field inhomogeneity. Ikegami and Crawford[5] have also made measurements of radiation near cyclotron harmonics for a beam-generated mercury-vapor plasma in a magnetic field produced by Helmholtz coils. The mechanism for producing transverse beam energy

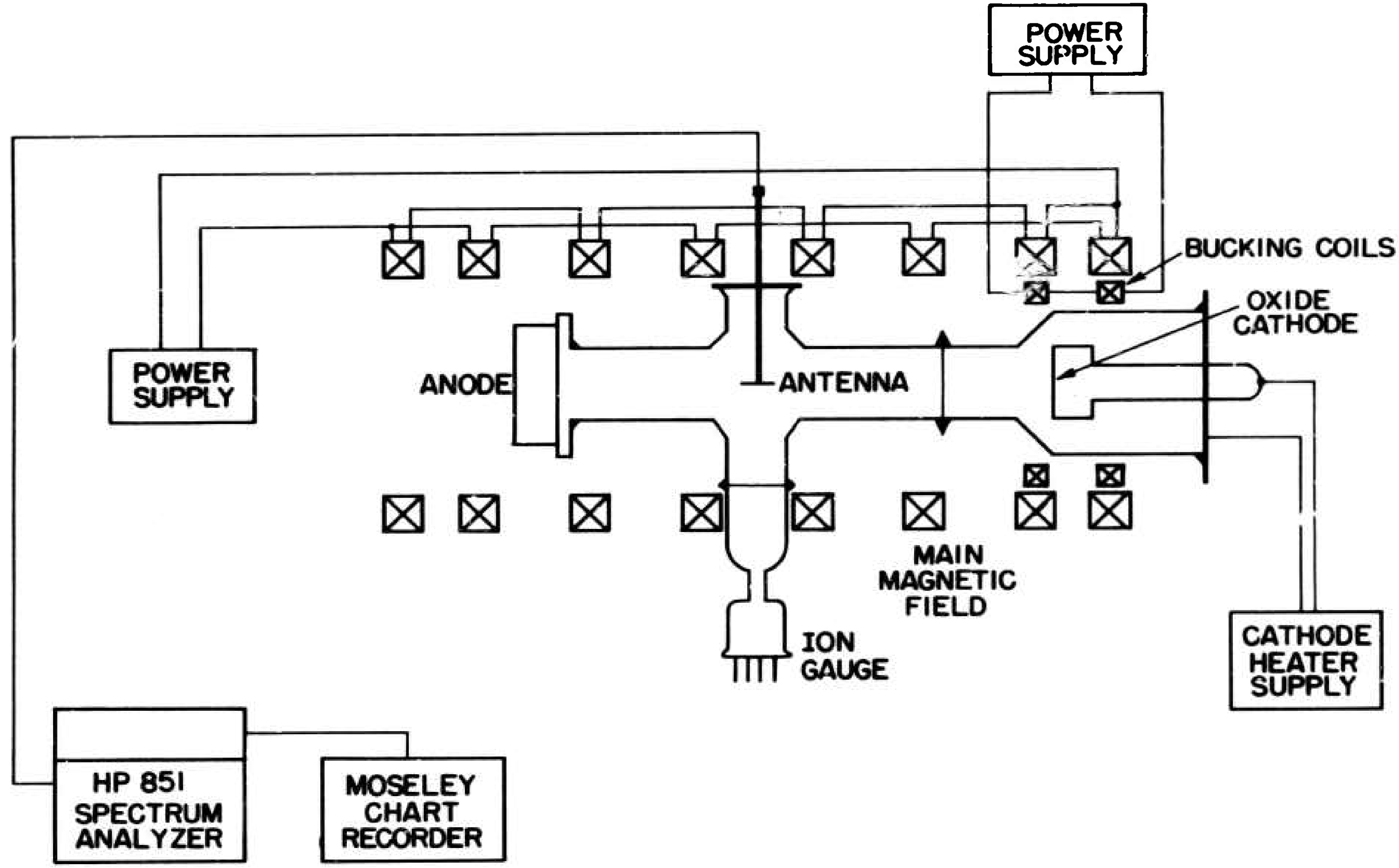

FIG. 5. Experimental set-up for study of cyclotron harmonic wave amplification.

was again an inhomogeneous magnetic field

During the reporting period, we have been studying radiation from an argon positive column discharge immersed in the inhomogeneous magnetic field produced by the set-up of Fig. 5. Use of the broad-band spectrum analyzer (HP 851) greatly simplifies the radiation measurements since it eliminates the necessity of sweeping the magnetic field. Typical data are shown in Fig. 6(a) for the signal received at the probe. The cyclotron frequency is 530 MHz at the probe and $\sim$ 3 MHz at the cathode surface. Certain peaks in the spectrum varied in frequency with changes in the discharge current. For example, analysis of the variation of Peak A shows that it varies with the square root of the discharge current and corresponds to the electron plasma frequency (see Fig. 6(b)). It is probably excited by beam/plasma interaction in an axisymmetric mode. Peaks B and C follow more complicated variations which are currently being elucidated.

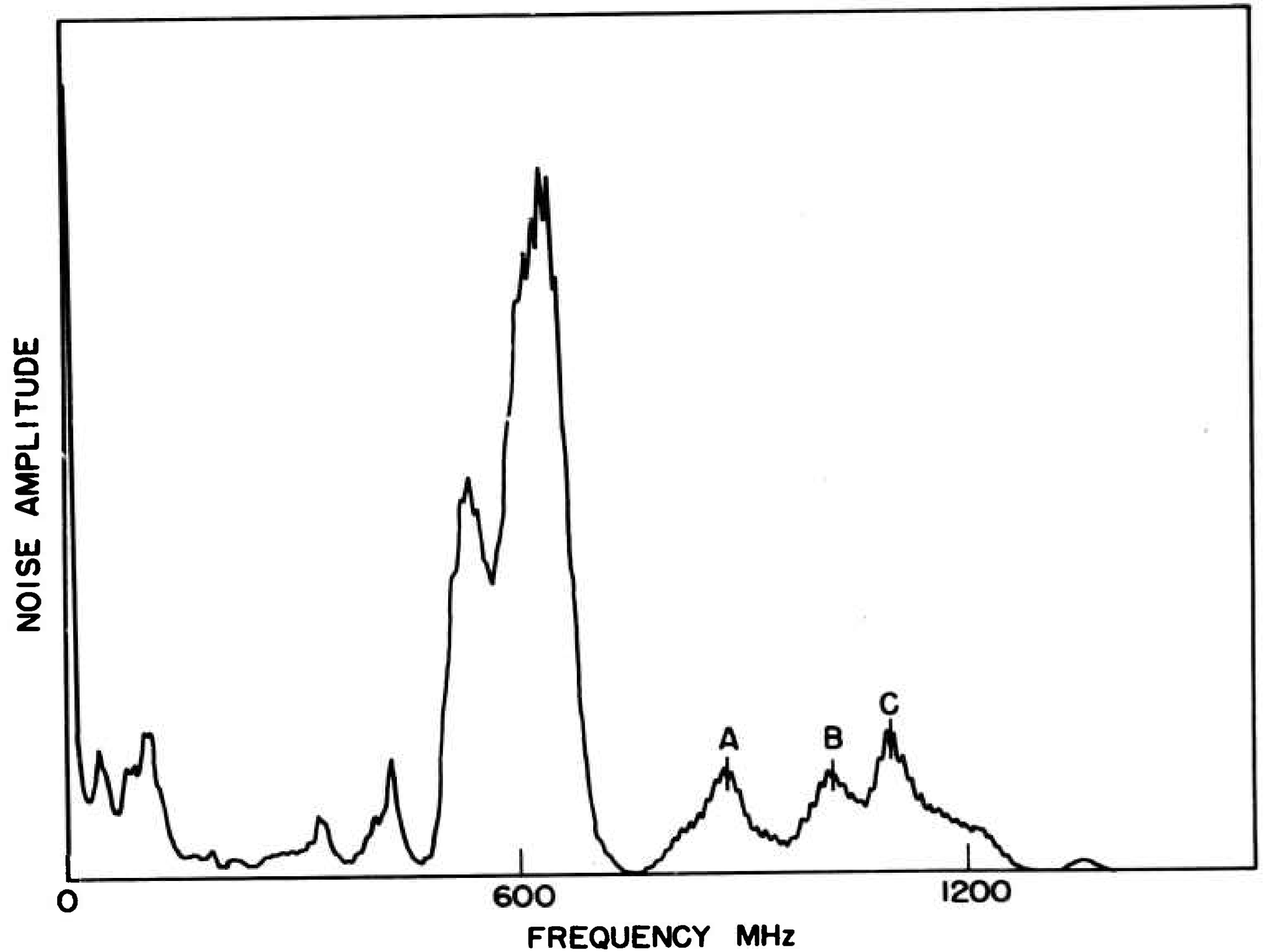

FIG. 6. Noise radiation from a magnetoplasma:

(a) Spectral distribution of the noise.

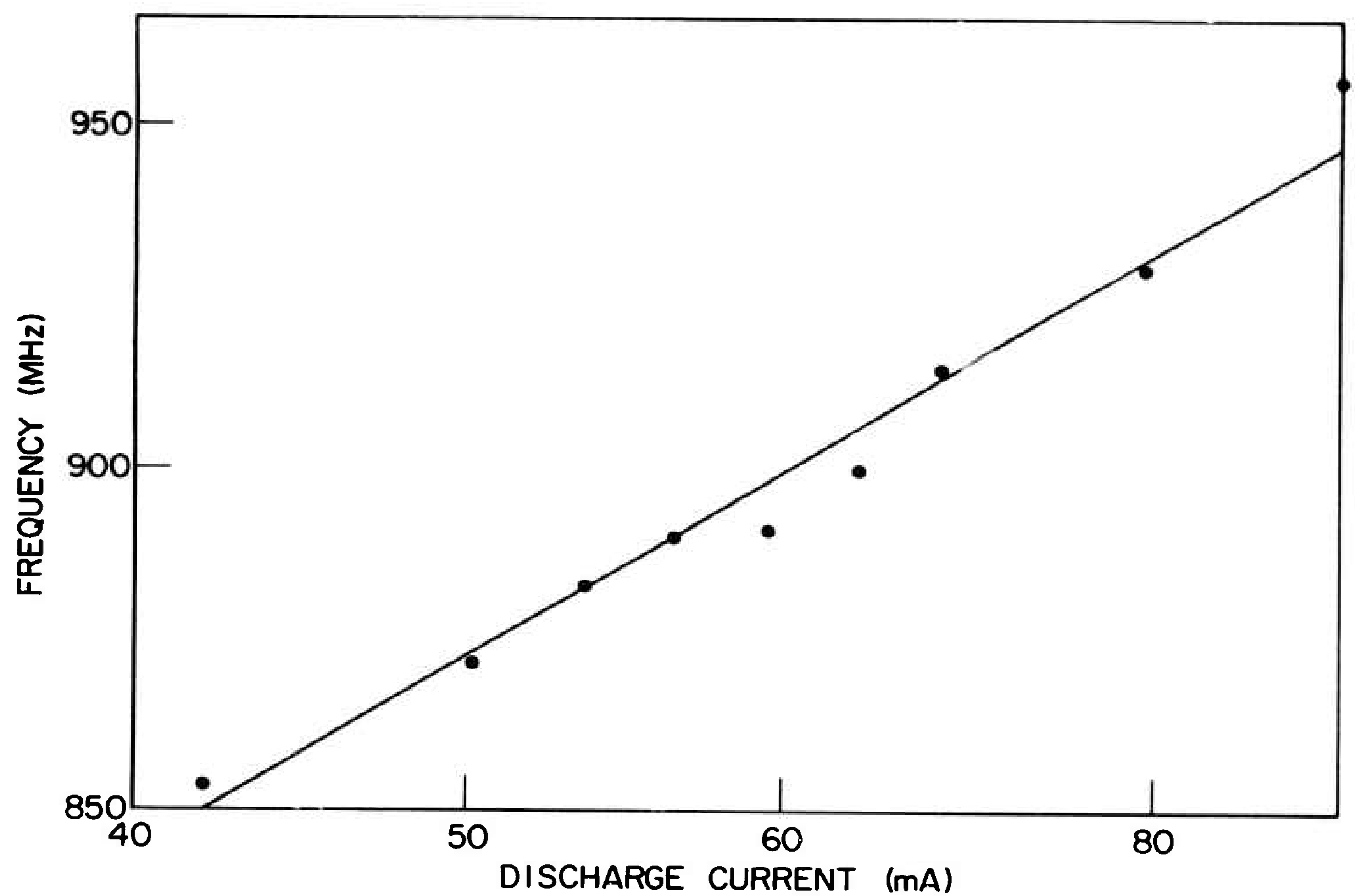

FIG. 6. Noise radiation from a magnetoplasma:

(b) Variation of Peak A with current [Current is plotted to a square root scale].

IV. ELECTROMAGNETIC WAVE AMPLIFICATION IN MAGNETOPLASMAS

In the absence of a static magnetic field, interaction of an electron beam with a plasma leads only to electrostatic beam/plasma interactions of the types described in Section II. When a static magnetic field is present, there are additional possibilities of electromagnetic wave interaction. That of special importance under the present contract is the interaction with the right-hand polarized electromagnetic wave known in ionosphere terminology as the "whistler" mode. It has been demonstrated theoretically that under conditions where a beam with transverse energy interacts with the plasma, wave growth in this mode should be possible, and that experimental situations in which this dominates over the electrostatic growth mechanisms occurring at the same time appear to be realizable.[6]

Comparatively little experimental work has been reported so far on propagation of the whistler mode in laboratory plasmas, and none of this seems to have been directed towards observation of wave growth due to interaction with a gyrating electron stream. Such a demonstration forms the primary object of this project. If growth in the whistler mode could be demonstrated, and utilized, it would offer very attractive practical features. In particular, coupling should be facilitated, since the amplification occurs in an electromagnetic mode, i.e., without conversion to an electrostatic mode.

The aims of the present project are as follows: First, to elucidate the theory of the whistler-type instabilities in the simplest geometry, and then to extend this to more realistic physical conditions, and second to demonstrate directly by experiment that growth can occur in this mode.

(A) Theoretical Studies.

In QR 4,[1] a set of numerical solutions for the simultaneous equations $D(\omega,k_{\parallel}) = 0$, and $\partial D(\omega,k_{\parallel})/\partial k_{\parallel}) = 0$ were presented with the beam velocity, $v_{0\parallel}$, and the transverse speed of the beam particles, $v_{0\perp}$ as parameters. The limiting values as $v_{0\parallel}$ approaches zero have been investigated analytically during this quarter, with the conclusion

that the branch-point locus should approach continuously the values corresponding to $v_{\parallel 0} = 0$, that is

$$\omega = \omega_c \pm i \frac{\omega_b v_{0\perp}}{2^{1/2} c} , \qquad (6)$$

as $v_{0\parallel}$ decreases. This continuity is shown in Fig. 7. The case in this limit was studied by Sudan.[7] He also concluded that an absolute instability occurs. For a practical laboratory beam/plasma interaction, the beam density will be small. The locus of branch points in the complex-ω plane, as $(\omega_b/\omega_c)^2$ decreases, is shown in Fig. 8. The corresponding saddle-point plot is given in Fig. 9. For most of the range studied, as the beam density decreases, the temporal growth rate increases, while the spatial (convective) growth decreases.

It was shown in a previous QR[8] that either collisions or non-zero plasma electron temperature can stabilize the absolute instabilities. The convective growth rate variation with ω_p , ω_b , $v_{0\perp}$, and $v_{0\parallel}$ as parameters was studied in QR 4. The limiting case for $v_{0\parallel}$ approaching zero has been analyzed during this quarter. For small $v_{0\parallel}$, the spatially-growing backward wave, primarily associated with the beam, is given approximately by,

$$v_{0\parallel} k_{\parallel} \approx \omega - \omega_c \pm i \frac{\omega_b v_{\perp}}{2^{1/2} c} . \qquad (7)$$

It is interesting to note that when $v_{0\parallel} = 0$, Eq. (7) becomes identical to Eq. (6). This expression has been checked with exact numerical computer solutions, and excellent agreement is obtained for $\omega \leq 0,8\, \omega_c$, in Fig. 10. As the beam velocity approaches zero, the two eam modes tend to infinity. The optimum beam velocity should be determined by collisions or temperature. This quantity will be established during next quarter.

It has been pointed out frequently that our present theoretical efforts are directed towards study of idealized models of beam/plasma interaction, and the motivation for studying what becomes of the plane whistler wave as plasma temperature and inhomogeneity make themselves

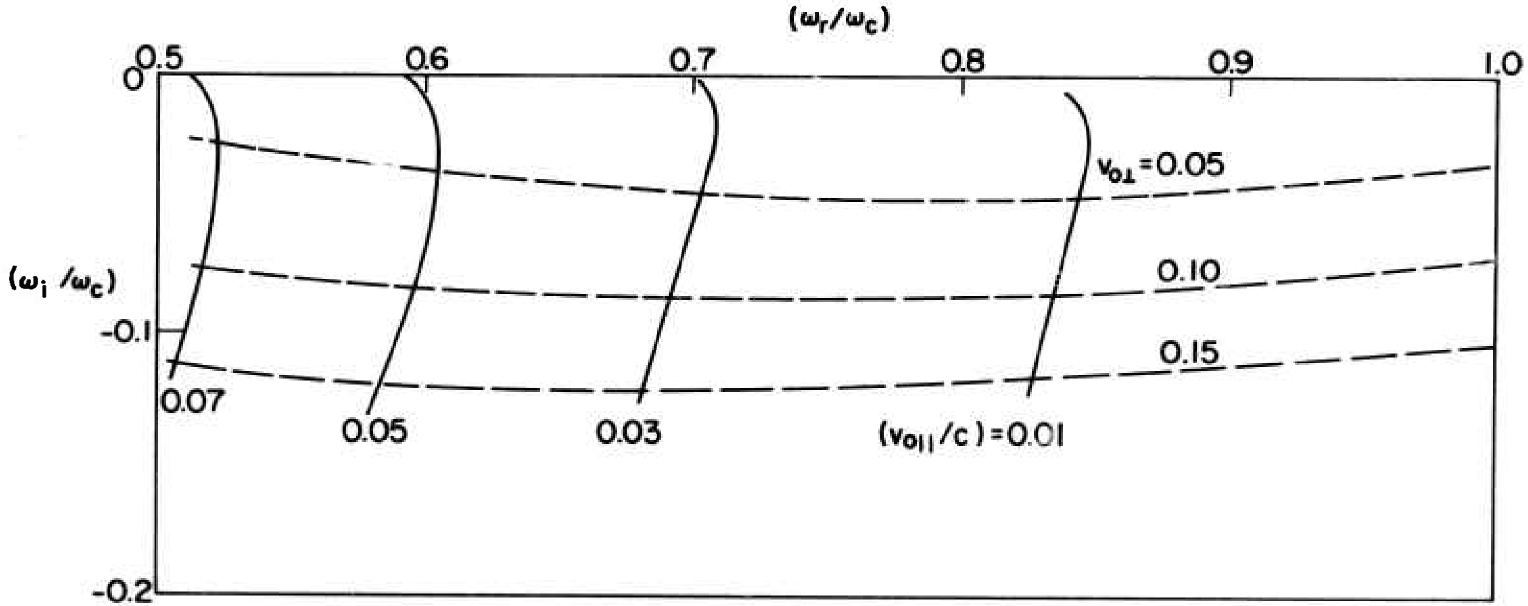

FIG. 7 **Whistler stability studies: Loci of branch-points in the complex-ω plane** $[(\omega_b^2/\omega_c^2) = 25$, $(\omega_b^2/\omega_c^2) = 1$, $(\nu/\omega_c) = 0]$.

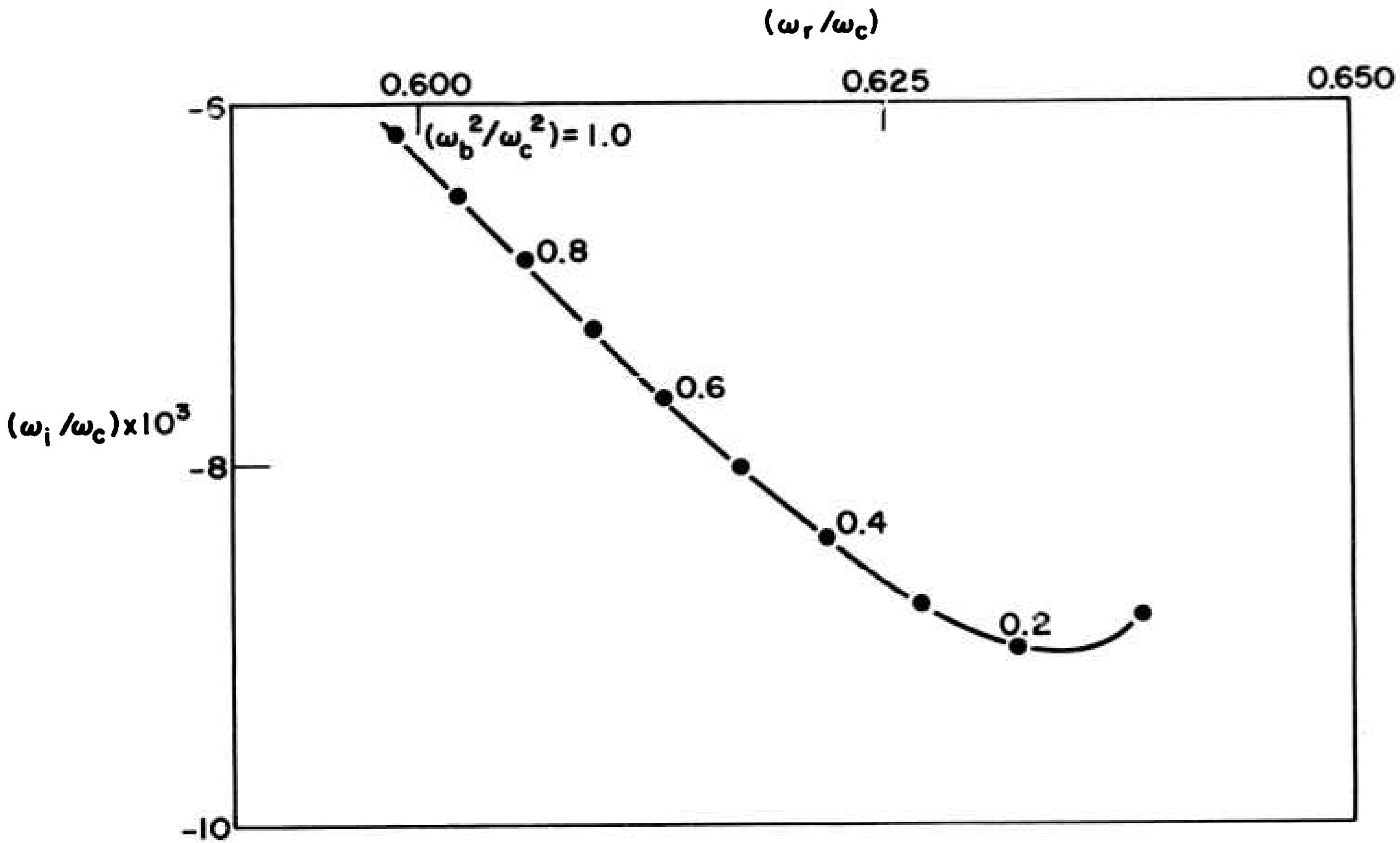

FIG. 8 Whistler stability studies: Locus of branch-point with varying (ω_b^2/ω_c^2) $[(\omega_p^2/\omega_c^2) = 25$, $(v_{0\parallel}/c) = -0.05$, $(v_{0\perp}/c) = 0.025]$.

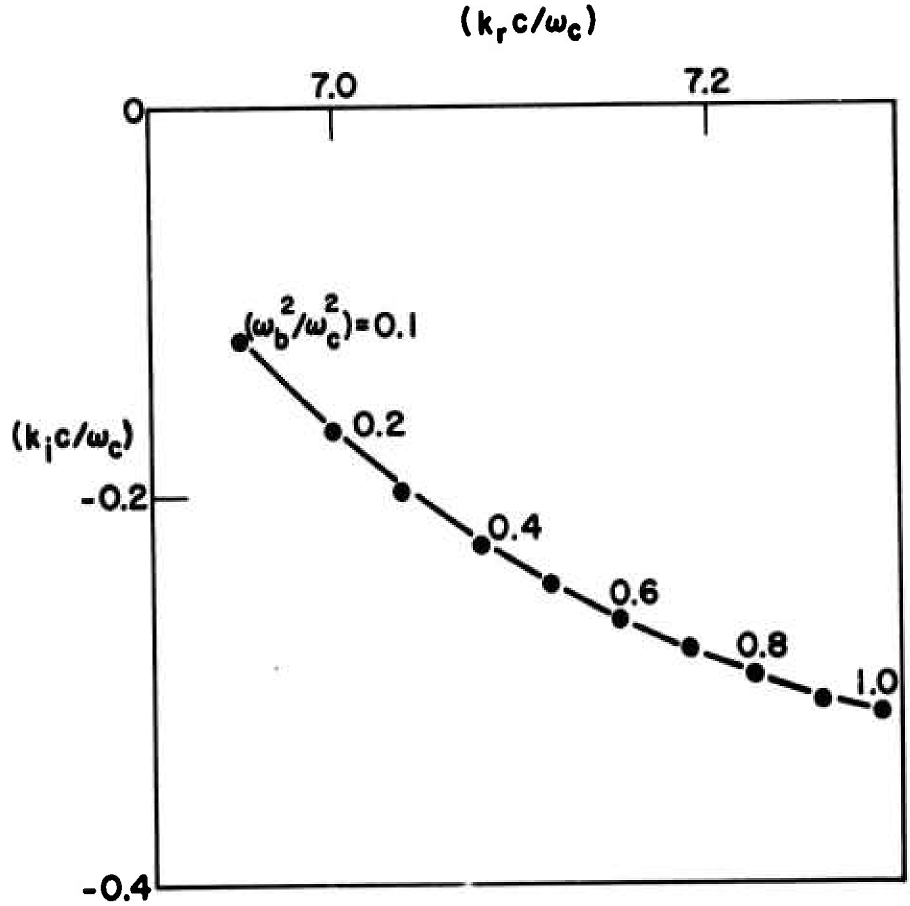

FIG. 9 Whistler stability studies: Locus of saddle-point in the complex-k plane with varying (ω_b^2/ω_c^2) $[(\omega_p^2/\omega_c^2) = 25$, $(v_{0_\parallel}/c = -0.05$, $(v_{0\perp}/c) = 0.025]$.

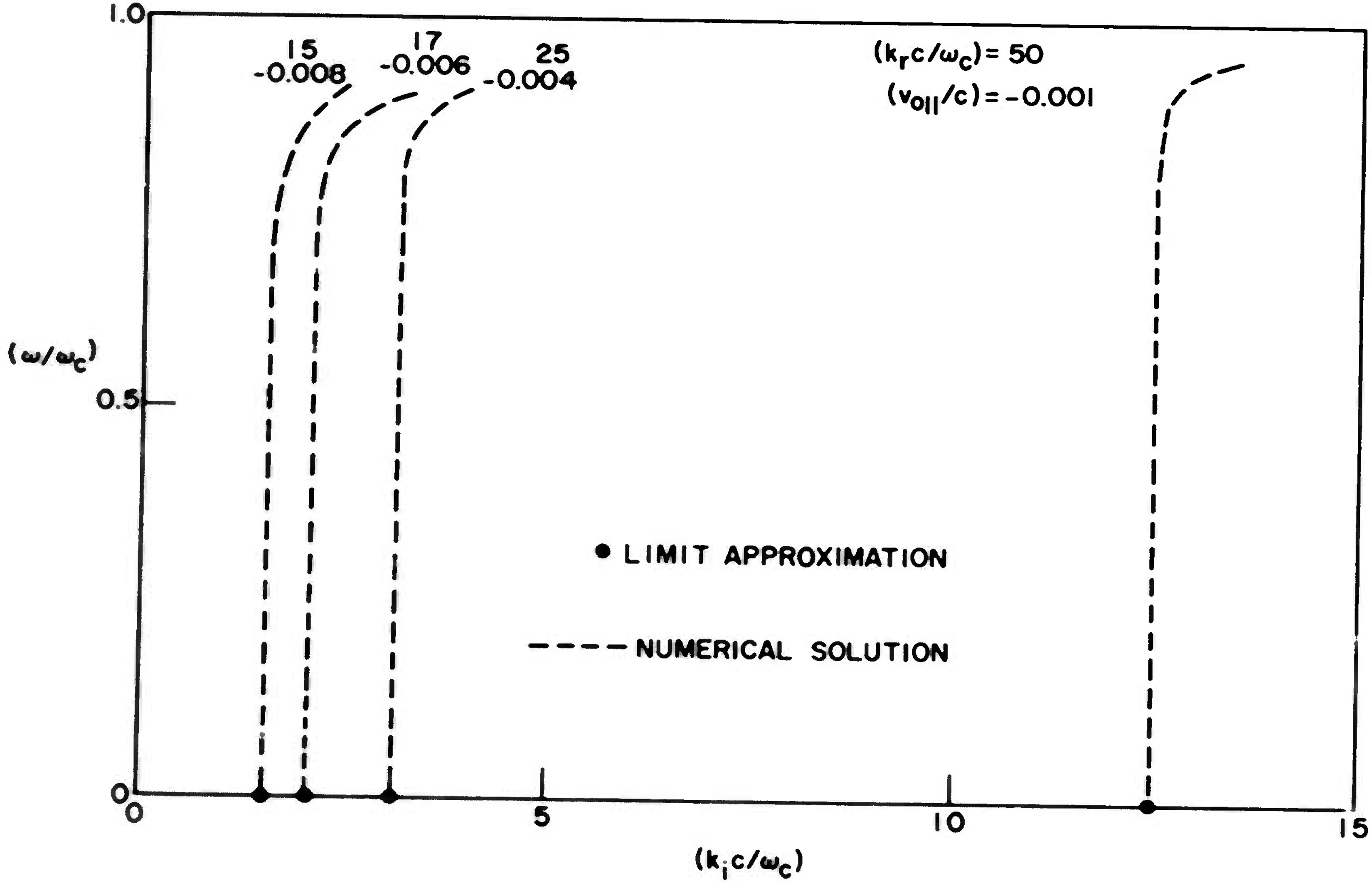

FIG. 10 Whistler stability studies: Convective amplification $[(\omega_p^2/\omega_c^2) = 25\ ,\ (\omega_b^2/\omega_c^2) = 0.5\ ,(v_{0\perp}/c) = 0.025]$.

felt was briefly outlined in QR 4.[1] Although the experimental work strives to simulate the conditions under which the simple plane wave theory is valid, the thermal spread and nonuniformity of the electron distribution will undoubtedly affect the results. In simplest terms, it is to be expected that the desired whistler is itself modified thereby and that additional wave types are simultaneously excited and coupled to the whistler mode. It has accordingly been considered important to develop the theoretical foundations for the analysis of wave propagation and stability in a hot, magnetized, inhomogeneous plasma.

It was explained in QR 4 that this task involves the combination of the Boltzmann equation for the distribution with Maxwell's equations for the electromagnetic fields. A viable method was expounded for extracting the electromagnetic sources from the perturbations of an equilibrium distribution, by reformulating the system description in terms of inverse velocity space.[9] The task that remained was to combine the resulting expression for the sources in terms of the fields with the field equations themselves.

During this quarter, that task has been carried out in a formal way. Whereas it had been envisaged that the combination would result in a partial differential equation of appropriately high order, however, recognition has now been given to the fact that a valid description of the system perturbations in the inhomogeneous case must involve an integral equation, rather than the differential type. This is a direct result of particle transport in the plasma and is vital to the stability problem because it is the interaction of the perturbing wave with classes of particles that are transported in resonance with the disturbance that must be expected to give rise to wave growth. The resonance phenomenon shows up as singularities in the integral equation, but it has been found possible to eliminate that difficulty, again by relying on an inverse velocity space description. The formalism developed for this analysis has been reported in detail in a technical report by P. Diament which is at present in the final stages of preparation. The basic results will be outlined here.

The hot, inhomogeneous system is described by its equilibrium "inverse phase space spectrum", $F_0(\theta)$, which is just the six-dimensional Fourier transform of the distribution function; θ is the position vector in inverse phase space. The force fields to which the particles are subjected, including the applied magnetic field and the internal ambipolar fields, are assumed to be describable by a constant matrix of proportionality to the phase, Y . Then the Laplace transform of the time development of a weak disturbance of the spectrum can be shown to be given by

$$F_1(\theta,s) = i\underset{\sim}{L}_s \int F_0(\theta_t - \theta_0)(\theta_t - \theta_0)\cdot A_6(\theta_0,s)\,d^6\theta_0/(2\pi)^6 \quad , \tag{8}$$

where $\underset{\sim}{L}_s$ denotes Laplace transformation, $A_6(\theta,s)$ incorporates the perturbing acceleration field and $\theta_t = \theta \exp Yt$ describes the unperturbed orbit in inverse phase space. This is the solution of the Boltzmann equation in terms of the perturbation A_6 . The latter is then obtainable by evaluating $F_1(\theta,s)$ and its gradient at the origin in inverse velocity space, $\underset{\sim}{\Lambda} = 0$. The combination is expressible by the pair of equations

$$\underset{\sim}{\alpha}(\underset{\sim}{k},s) = \frac{i\omega_0^{\,2}}{s^2 + k^2c^2}\left(c^2\underset{\sim}{k} + f\frac{\partial}{\partial\underset{\sim}{\Lambda}}\right)F_1(\underset{\sim}{k},\Lambda,s) \quad , \tag{9}$$

$$F_1(\underset{\sim}{k},\underset{\sim}{\Lambda},s) = i\underset{\sim}{L}_s \int \left[F_0\underset{\sim}{\Lambda}_t\cdot\underset{\sim}{\alpha}(\underset{\sim}{k}_0,s) + \underset{\sim}{G}_0\cdot\frac{\underset{\sim}{k}_0}{s}\times\underset{\sim}{\alpha}(\underset{\sim}{k}_0,s)\right]\frac{d^3k_0}{(2\pi)^3} \quad , \tag{10}$$

where $\underset{\sim}{G}_0 = \underset{\sim}{\Lambda} \times \partial F_0/\partial\underset{\sim}{\Lambda}$; the arguments of F_0 and $\underset{\sim}{G}_0$ are $(\underset{\sim}{k}_t - \underset{\sim}{k}_0,\underset{\sim}{\Lambda}_t)$, where $\theta_t = (\underset{\sim}{k}_t,\underset{\sim}{\Lambda}_t) = (\underset{\sim}{k},\underset{\sim}{\Lambda}) \exp Yt$, and ω_0 and $\underset{\sim}{\alpha}$ are essentially the plasma frequency and the perturbing electric field. This pair constitutes an integral equation for the electric field, with the plasma frequency as the eigenvalue.

These results have been specialized to the case of a uniform magnetized plasma column with a Maxwellian distribution in velocity and a

Gaussian spatial profile. The final integral equation can have either a Gaussian, or exponential, or Bessel function kernel and requires routine numerical analysis to extract the dispersion relation and stability criteria. This is planned, for the whistler mode, for the next quarter. Preliminary results already indicate that one effect of inhomogeneity is to shift resonances away from the cyclotron frequency by an increment which depends on the time scale of traverse of a thermal particle across a significant region of inhomogeneity.

(B) Experimental Studies.

During the quarter, whistler experiments have continued on the S-band set-up using a pulsed reflex discharge. Study of standing waves indicates that the waves observed a few hundreds of microseconds after the peak of the current pulse should be classified as whistlers. These waves make smooth transitions to free space electromagnetic waves at low plasma densities. When plasma density is low, however, the free space wavelength for an L-band signal ($\sim$ 20 cm) is too great compared with the plasma column cross-section (7.5 cm diameter) to expect simple plane wave theory to apply. Efforts have consequently been made to detect whistlers at higher electron densities. At such densities, whistlers have been observed damped due to collisions. Quantitative measurements of their propagation have been obtained as follows:

The waves are excited by short electric dipoles perpendicular to the magnetic field lines. A sampling time in the afterglow when the plasma density is reasonably uniform is chosen, and interferograms are obtained by applying a fixed frequency and beating the output signal against a component derived from this. Typical probe measurements of the evolution of the plasma profile, made using a movable probe, are shown in Fig. 11. It is clear that the density is substantially uniform for times greater than 200 μs. Figure 12 shows some interferograms obtained at 1.2ms. Since the measured wavelengths at $\sim$ 2.0 GHz are of the order of 1 cm, plane wave theory should be closely approximated.

Data similar to that of Fig. 12 serve to give the whistler dispersion characteristics. These are shown in Fig. 13, and may be compared with plane wave theory. To do so, requires knowledge of the electron

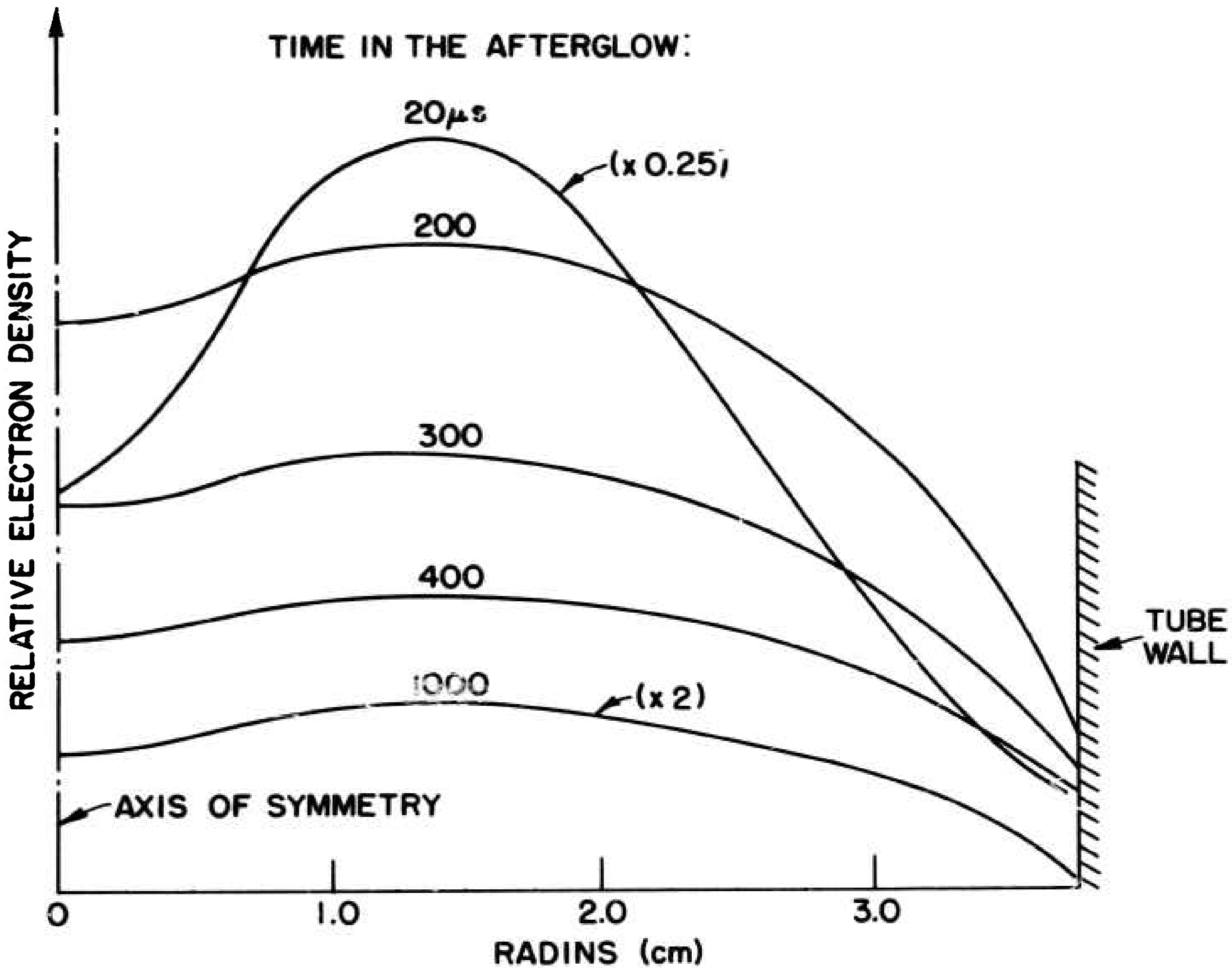

FIG. 11 Pulsed reflex discharge: Variation of electron density profile in the afterglow.

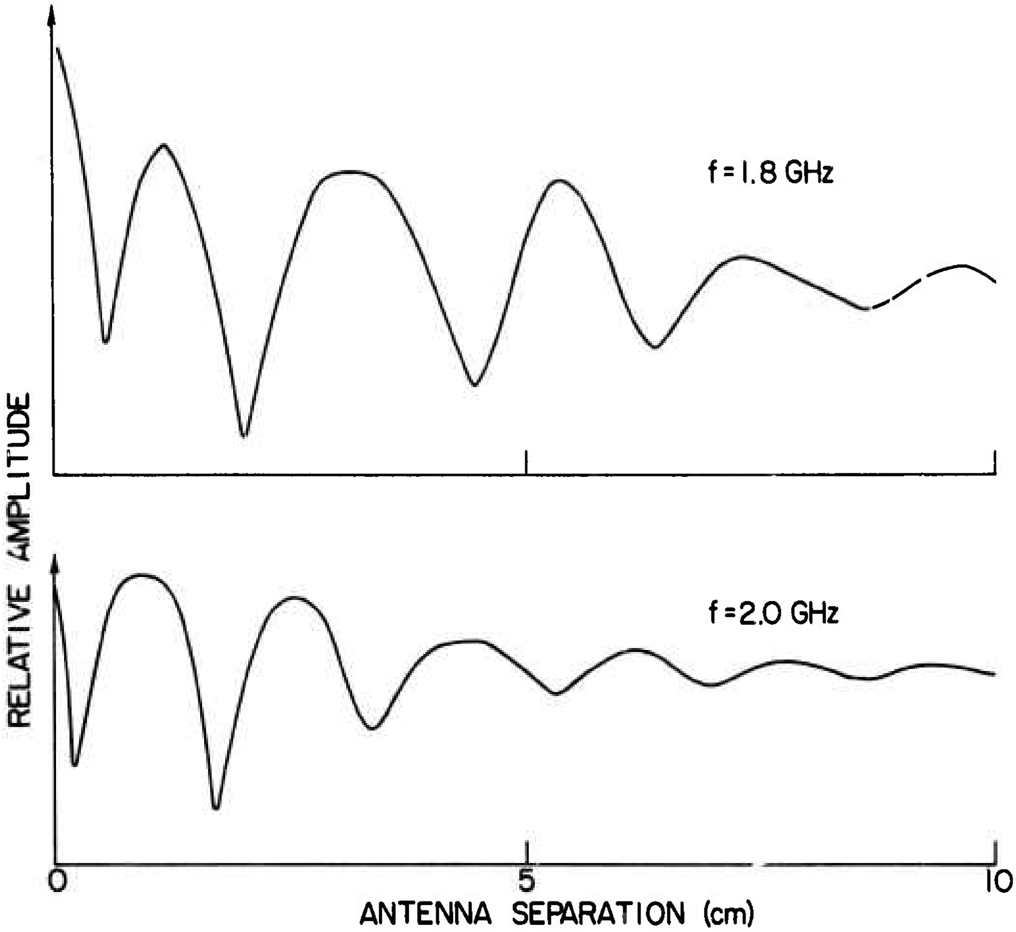

FIG. 12. Whistler propagation characteristics (f_c = 2.3 GHz , $f_p \approx$ 14 GHz) .

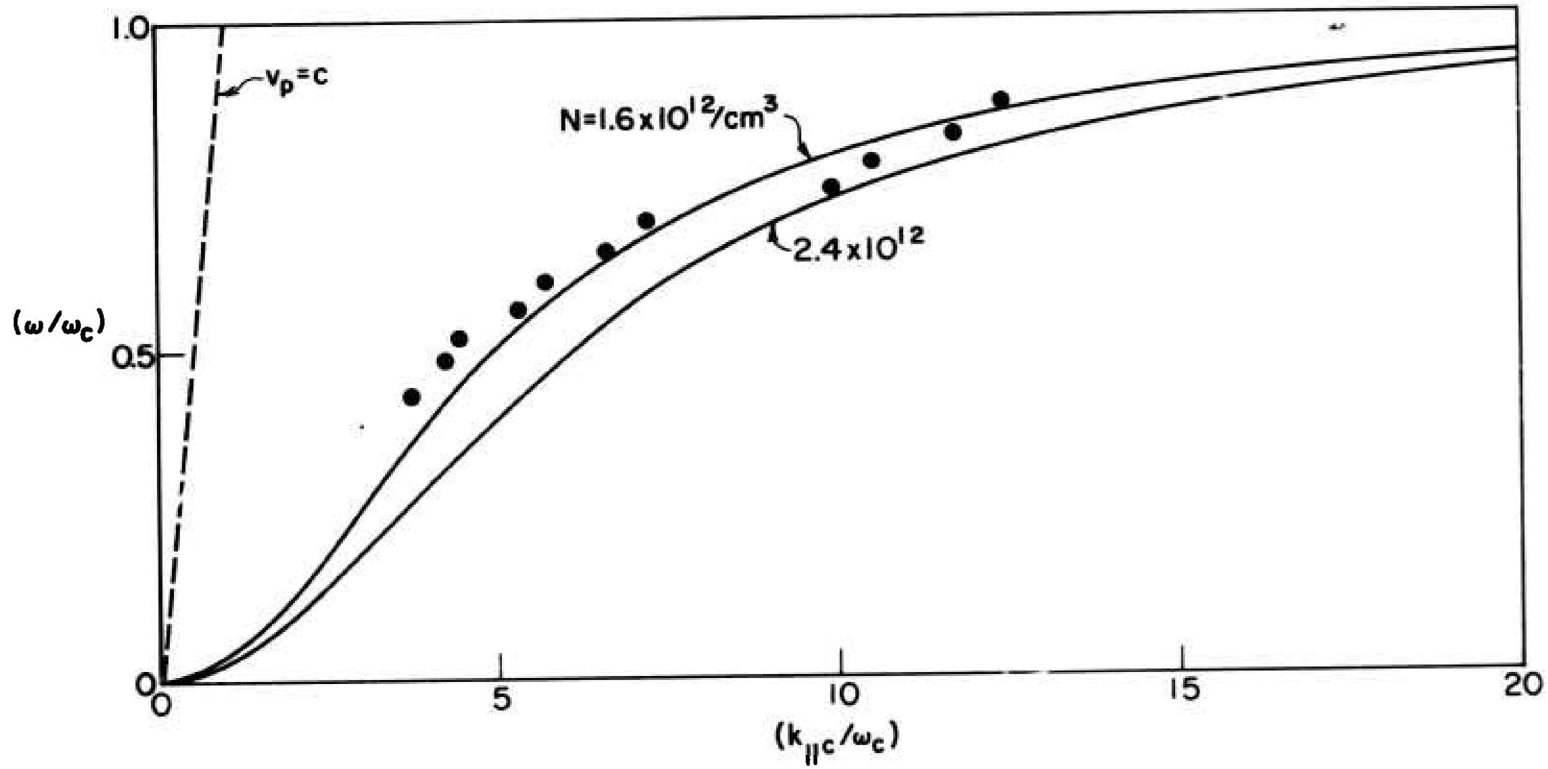

FIG. 13 **Whistler propagation: Dispersion characteristics**
$(f_c = 2.3$ GHz , $f_p \approx 14$ GHz) .

density. This was obtained by calibrating the Langmuir probe measurements by means of a K-band interferometer. Good agreement is indicated in the figure for the range covered by the measurements. To extend them to lower (ω/ω_c), higher magnetic fields are needed. Further measurements will be made next quarter in the new magnetic field system, which gives cyclotron frequencies up to K-band. The status of this set-up is as follows. The magnet system that has delayed this project has been tested and is now available. It is shown in Fig. 14. It can provide a steady field of up to 7.7 kGauss which is uniform to ± 0.25% over a distance of 40 cm. The maximum AC ripple of the magnetic field is less than 1%, and the ripple at full power is less than 0.1%. The magnet dissipates nearly 150 kW at maximum power.

A vacuum system (not shown in figure) has been assembled, and experiments now under way in the set-up are directed towards obtaining a high-density rf discharge at a few MHz. The rf transmitter that will be used to produce the plasma is in the final testing stages. The various probes and interferometers which will be used as diagnostic devices have already been constructed. The discharge will be produced by the rf transmitter, operated in a pulsed mode, and allowed to decay. A maximum plasma density of approximately $10^{13}/cm^3$ is expected.

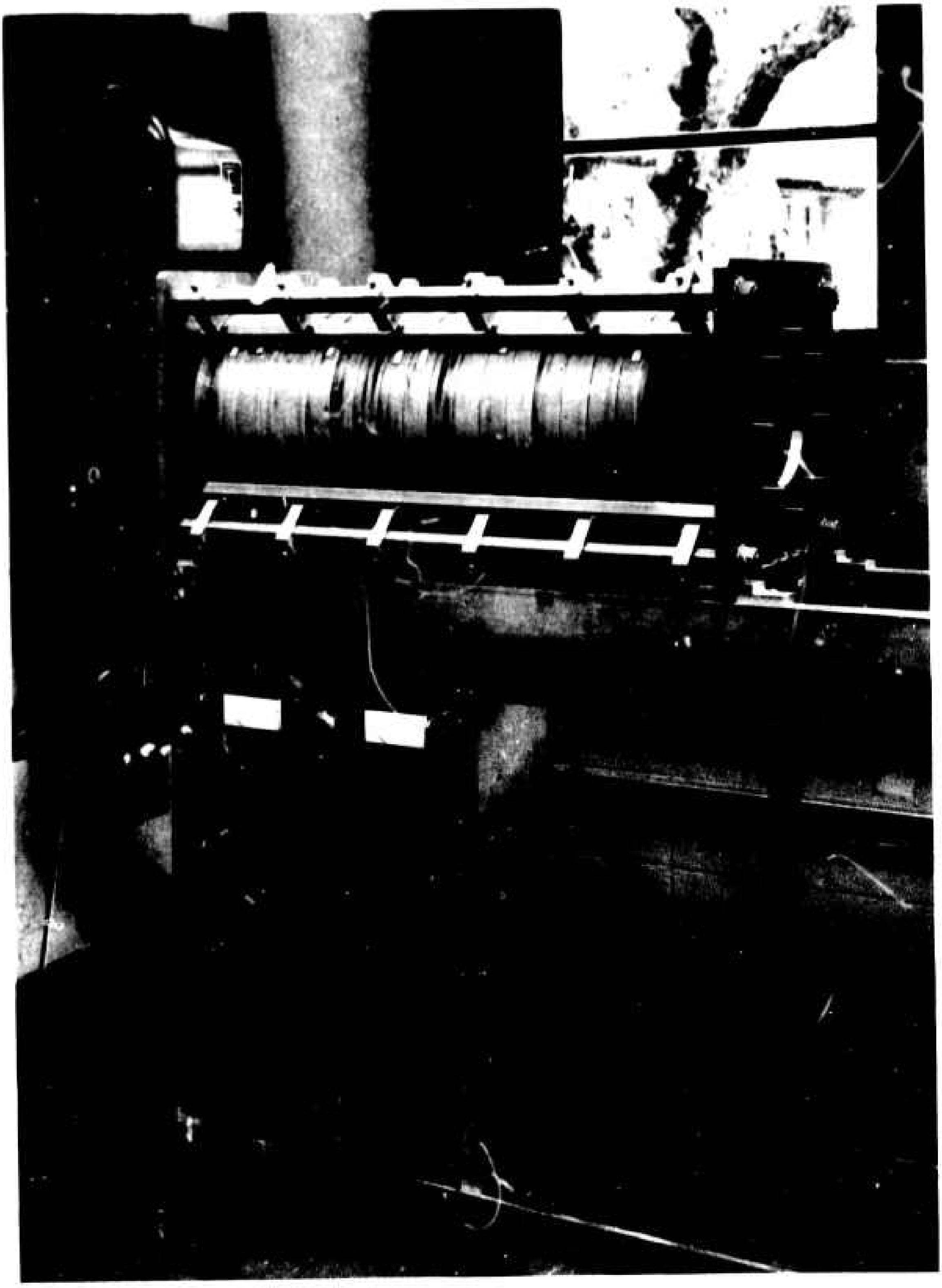

FIG. 14 K-band magnetic field system for use in whistler amplification studies.

V. FUTURE PROGRAM

Most of the details of our program for the coming quarter have been dealt with in the relevant theoretical and experimental sub-sections of Sections II-IV. Summarizing, the program is as follows:

(i) Beam/plasma amplification with transverse modulation -- Theoretical work will continue on beam/surface wave interactions, first for the $m = 0$ mode, to check the experimental results already obtained, then for the $m = 1$ mode. It is anticipated that our measurements on the sealed-off tubes constructed so far will be completed, and that further studies will be made in a more flexible continuously-pumped system.

(ii) Electrostatic wave amplification in magnetoplasmas -- Further measurements on the noise spectrum due to magnetoplasma wave excitation by electrons with transverse energy will be made, first with a view to identifying the various frequencies so far observed, then with the aim of verifying the theory quantitatively for a delta-function beam interacting with a cold plasma.

(iii) Electromagnetic wave amplification in magnetoplasmas -- Our studies of the relevant dispersion relations will be extended numerically. It is hoped that the propagation measurements will be completed in the S-band magnetic field system. They will then be extended in the K-band system preparatory to introducing an electron beam with transverse energy to excite wave growth.

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13 ABSTRACT

This report describes a program of work on beam/plasma interaction. Both electrostatic and electromagnetic wave amplifying mechanisms are under investigation. For the former, studies in the absence of a static magnetic field are directed towards verifying the theory for the cases of finite beam/infinite plasma and beam/surface wave amplification, when transverse modulation is applied. A dipole resonant coupling system for such interactions is under study. Two distinctly different lines are being followed for interactions in the presence of a static magnetic field: Electrostatic cyclotron harmonic wave interaction is being examined, both theoretically and experimentally, and the potentialities of electromagnetic wave growth in the "whistler" mode are being investigated.

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